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DIVISION OF WATER RESOURCES & HYDRAULICS
DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF CINCINNATI
CINCINNATI, OHIO

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GROUND WATER BASIN DYNAMICS

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by
Herbert C. Preul
and
Louis M. Laushey

SEPTEMBER, 1968

Project No. Ohio State W-116
Submitted to:
Water Resources Center
The Ohio State University

and to:
Office of Water Resources Research
U. S. Department of Interior

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A number of graduate students were supported under this project or made a major contribution to it. Their service is gratefully acknowledged, and their names are listed below:

L. V. Popat
P. K. Prabakhar
M. T. Lee
R. C. Lewis
Y. J. Tsai

SUMMARY

GROUND WATER BASIN DYNAMICS

The project was divided into five studies which fit together to concentrate on the general subject of Ground Water Basin Dynamics. Each of the studies culminated in the writing of a thesis used for fulfillment of requirements for the degree of Master of Science by graduate students in the Division of Water Resources and Hydraulics, Department of Civil Engineering at the University of Cincinnati. The studies were carried out under the supervision of the principal investigators, Dr. Herbert C. Preul and Dr. Louis M. Laushey. Listed below are the individual subjects and the graduate students involved.

Basic Studies

1. "A Study of Pore Distribution and Permeability in Turbulent and Laminar Flow," M. T. Lee
2. "Darcy's Law During Unsteady Flow," L. V. Popat
3. "Unsteady Ground Water Flow," Y. J. Tsai

Mathematical Model Studies

4. "Mathematical Model of the Big Bend Well Field," R. C. Lewis
5. "Mathematical Model for the Ground Water Basin of the Great Miami River," P. K. Prabakhar

Abstracts for all of the above studies are given on following pages. Because of the lengthy dissertations written on each of the above subjects, detailed information is given only for the study "Mathematical Model for the Ground Water Basin of the Great Miami River." This study represents the most generally applicable results of the project.

Detailed information on the results from the other studies will be made available upon request.

Two publications have resulted from the project to date. They are: "Effect of River Water Quality on an Adjacent Aquifer" by Herbert C. Preul and L. V. Popat, presented by Herbert C. Preul at Third Annual Symposium, Water Resources Center of the Ohio State University, September, 1967; printed in Proceedings of the Symposium.

"Darcy's Law During Unsteady Flow" by Louis M. Laushey and L. V. Popat, presented by Louis M. Laushey at Congress of the International Union of Geodesy and Geophysics, International Association for Scientific Hydrology, Bern, Switzerland, September, 1967; printed in Proceedings of the meeting.

ABSTRACT

A STUDY OF PORE DISTRIBUTION AND PERMEABILITY IN TURBULENT AND LAMINAR FLOW

This is a study of the relationship between permeability and mean pore area of porous media. Experiments were carried out using water flow through a permeameter packed with glass beads and with sand. Results were compared with previous data by other investigators.

A formula was developed based on the results which is representative of the mean pore area in a media such as a sand. A correlation between permeability and mean pore area in sand was found to approximate a linear relationship. With some additional verification, it is anticipated that these results may be useful for ground water flow determinations.

ABSTRACT

DARCY'S LAW DURING UNSTEADY FLOW

Darcy's Law specifies the proportionality of the velocity to the piezometric slope for steady laminar flow through porous media. Many experimenters have verified Darcy's Law for many ground water flow phenomena.

The proportionality of the instantaneous velocity to the instantaneous piezometric slope is often assumed to be valid during unsteady flow too. That Darcy's Law can be extended to unsteady flow apparently has never been verified. It is shown that the velocity and piezometric slope are not always proportional during unsteady flow.

The study is concerned only with flow phenomena for which Darcy's Law was found to be correct for steady flow. Nevertheless, it is relevant to review the research which has disclosed some factors which deny Darcy's Law for even steady flow. The experiments to be reported purposely avoided all of the following situations of steady flow, non-Darcy behavior reported by previous researchers.

Darcy's Law is shown to be incomplete for laminar unsteady flow through porous media. Experiments and theory indicate that the total derivative of the water table elevation, with both distance and time, must be used for the gradient of the static head.

Corrections to Darcy's Law are required when $\delta/\delta X\ (\delta h/\delta t)$ and $\delta/\delta t\ (\delta h/\delta t)$ are not zero. Darcy's Law is correct for the quasi steady state of $\delta h/\delta t$ not zero if the two previously stated derivatives are zero.

Experiments seem to confirm that

$$V = -K\frac{dh}{dX} = -K\left\{\frac{\partial h}{\partial X} + \frac{\partial h}{\partial t}\frac{dt}{dX}\right\} = -K\left[\frac{\partial h}{\partial X} + \frac{\partial h}{\partial t}\;\frac{\frac{\partial}{\partial X}\frac{\partial h}{\partial t}}{\frac{\partial}{\partial t}\frac{\partial h}{\partial t}}\right]$$

ABSTRACT

UNSTEADY GROUND WATER FLOW

In this study, the characteristics of unsteady ground water flow were investigated by means of a viscous fluid model of the Hele-Shaw type. This type of model provided a convenient way of observing unsteady flow surfaces without the complications of an aquifer.

The model consisted of a plate glass channel, a head reservoir, an outlet control gate, and recording units. The fluid used for the experiments was crystal clear 96% glycerine. With the following boundary conditions for $h = h\ (x,t)$:

$$h\ (o,t) = o,\ t = o$$

$$\frac{\partial h\ (L,t)}{\partial x} = o,\ t \geq o$$

$$q\ (L,t) = o,\ t \geq o$$

$$\text{and } h\ (x,o) = H,\ o \leq x \leq L$$

a large variation of the water surface was created and yielded a large magnitude of unsteadiness.

One-sixteenth inch diameter small plastic particles were placed on the surface of the fluid in order to observe the path of the movement of water surface. All of the data were taken by photographs on slides at the predetermined time interval. The finite difference method and high speed digital computer have been adopted and used for analyzing the recorded data to obtain other useful terms.

If $F\ (x,\ y,\ z,\ t) = o$ is the function describing the phreatic surface and $P\ (x,y,z) = o$ is a particle on the surface at time t, the total derivative of the function with respect to t and x for the two dimensional

potential flow are found as

$$\frac{Dh}{Dt} = \frac{\delta h}{\delta t} + \frac{\delta h}{\delta x}\frac{dx}{dt} \tag{1}$$

and

$$\frac{Dh}{Dx} = \frac{\delta h}{\delta x} + \frac{\delta h}{\delta t}\frac{dt}{dx} \tag{2}$$

where $\frac{Dh}{Dt}$ and $\frac{\delta h}{\delta t}$ are the rate of vertical variation of the particle and water surface respectively. $\frac{Dh}{Dx}$ and $\frac{\delta h}{\delta x}$ are the slope of the particle path and water surface respectively. $\frac{dx}{dt}$ represents the velocity component in x-direction.

The results of the experiments showed both values of $\frac{dx}{dt}$ calculated from equations (1) and (2) are equal to each other for a particle at an instant time and also equal to the total derivative $\frac{dx}{dt}$. Therefore, the characteristics of the water surface moment can be represented by the derivative of a fluid particle on the surface.

The external resistant force in steady ground water flow can be shown as $F_f = \frac{Q\gamma}{K}$ where Q is the flow rate

γ is the unit weight of fluid

K is coefficient of permeability

F_f is resistant force in lbs. per unit length

while in unsteady flow the total resistant force is expressed by $F_f = \frac{b}{2}\int_{L_1}^{L_2} \frac{h^2}{x}\,dx$ for a reach of aquifer, L_1 to L_2 with basin width b.

A functional relation of the form $\frac{h}{H} = f\ (\frac{x}{L}, \frac{KH}{L^2m}\ t)$ containing only dimensionless parameters can be proved satisfactorily by plotting $\frac{h}{H}$ as a function of $\frac{x}{L}$ and $\frac{KH}{L^2m}$ t. The plot is independent of different experiments.

Outflow discharge, according to the continuity $q(t) = m \int_o^L \frac{\partial h(x,t)}{t} dx$, also can be unified all individual tests to one dimensionless outflow hydrograph with the relation $q^1 = \frac{mL}{KH^2} q$.

The ground water hydrograph in dimensionless plots is a very useful tool to estimate the base flow of a flood hydrograph for the condition of aquifer under consideration.

Notation Definitions:

h, Depth of ground water at time t and distance x from seepage face

H, Original depth of ground water level

L, Length of the aquifer

K, Coefficient of permeability

m, Storage coefficient

ABSTRACT

MATHEMATICAL MODEL OF THE BIG BEND WELL FIELD

1. Conclusions

This study had for its purpose the investigation of the Big Bend well field, located in the lower valley of the Great Miami River, by constructing a mathematical model that would relate inflow and outflow from the well field to river and ground water surface elevations. In this investigation, computed inflow and outflow were compared with actual metered ground water withdrawals from the well field. The findings of this study were:

a. Application of the conservation of mass formula to the Big Bend well field in the Great Miami Valley has resulted in a very close check of computed outflow with metered withdrawals from the well field in the period October, 1959 through October, 1967.

b. By choosing time periods where surface and ground water elevations were stable both at beginning and ending of the period, laminar flow equations were applied with little observed error. However, when the elevations of influent river were changing rapidly either at beginning or end of a time period, very erratic values of outflow resulted.

c. The natural infiltration rate of the river averaged about seven gallons per square foot of river bed per day in summer periods and about 13.5 gallons per square foot per day during winter periods. With heavy continued pumping, the ground water surface adjoining the river dropped until it was no longer in contact with the river; for example, in November, 1967 the ground water surface was drawn down nearly 14 feet below the river surface.

d. The slope of the ground water from the mound below the river to the pumped wells was very close to that necessary to satisfy Darcy's equation for flow through a porous medium under steady flow conditions.

e. When the beginning and ending ground water elevations for a particular time period were averaged, the computed outflow values were more accurate than when slopes were determined merely by subtracting end of period elevations from beginning values.

f. An analog model of the Big Bend well field simulated the relationships between storage, well level variations, and outflow, and it has been helpful as a demonstration model. However, the varying time scale of the model indicates the need of further refinements before it will yield quantitative values.

It is believed that this study has demonstrated that the close correlation between computed and measured outflows in the Big Bend well field, where much data are available, permits application of the method to a considerably larger ground water basin where aquifer parameters and influent stream flow data have been determined.

2. Computer Solutions

The difficulty of predicting ground water elevations at established observation wells for assumed future withdrawals employing manual computation of a mathematical model of a relatively small and well known area such as the Big Bend well field points up the necessity of solving area ground water balance problems by computer methods.

In comparing the relative advantages of the digital and the analog computers, one must consider the possible uses for a mathematical model of a well field. These may include:

a. Determining the optimum perennial yield of an established but incompletely utilized aquifer.

b. Predicting the yield of an undeveloped aquifer or portion of aquifer.

c. Finding the water table variations at selected observation points in an aquifer that result from different pumping patterns and/or rates.

d. Learning the effects both as to quantity and quality of artificial recharge by waters of varied chemical and bacteriological characteristics.

e. Studying the results of some change in the regime of the principal influent stream that flows over a ground water aquifer.

f. Demonstrating to lay groups the effects of various uses, artificial recharge, change in stream regime or area infiltration on an aquifer.

g. Arriving at the most efficient overall management of a complete water system of which wells are only one source of supply.

Assuming adaption of computers to answer the questions and objectives set forth above, a comparison of the advantages of employing each of the two principal computer types suggests:

3. Analog Computer

Use of the analog computer to solve well field problems provides certain unique advantages, some of which are:

a. Provides a scale model of the area involved with easily grasped similarity between electrical and hydraulic units and with results reported at the location at which they are desired. This then permits demonstration and explanation to the wide range of persons interested in water management such as public officials, public and private water system managers, bonding company or other financial experts, lawyers, engineers, students, and the interested lay people whose support of any public program is essential.

b. Permits adjustment of the aquifer parameters, points of withdrawal, locations of observation, change of infiltration with a continuous reporting of that adjustment on other variables in a quick, clear, easily understood manner.

c. Reports changes with relation to a time scale that maintains proper relationships between all variables and allows observation of changes while they are taking place in a manner that aids in understanding the functioning of the system.

4. Digital Computer

Employment of the digital computer in the solution of typical well field problems permits accomplishing the following objectives:

a. Requires minimum preparation time and expense. Once a program has been written and checked it can be used repeatedly for changing report data.

b. Permits great flexibility in problem solution and introduction of sophisticated corrections for well interference, well entrance loss, actual water surface slope, different values for vertical and horizontal formation permeability.

c. Readily applicable to other aquifers. The new area constants and aquifer parameters can be quickly inserted into the original program.

ABSTRACT

A MATHEMATICAL MODEL FOR THE GROUND WATER BASIN OF THE GREAT MIAMI RIVER

The objective of this research was to set up a mathematical model which would be generally representative of the ground water dynamics in the buried valley of the Great Miami River which is located in southwestern Ohio. After proper verification of the proposed mathematical model with actual field data, it is hoped that the model may eventually be used for purposes of management of the main ground water resources of the valley.

The method presented is intended to be a general method of analysis. Verification attempts using existing field data have shown that the model has limited accuracy as presently devised. Further refinement of the model will be required as additional field data become available.

A unique aspect of the model is that it has been developed for a basin which is long and very narrow. The main ground water resources lie along the Miami River and are roughly 90 miles in length and average approximately 2 miles in width. Because of this long and narrow basin configuration, it might seem that the ground water dynamics at one end of the valley would have little affect on portions remote from it. This may be true under conditions of low ground water extraction. But as extraction increases, one location will affect remote locations because of the hydrogeologic connection of the water bearing aquifers and because the Miami River itself is known from previous studies to be the major source of ground water recharge in the valley. A good mathematical model, therefore, should provide a tool by which the operating agency, the Miami Conservancy District in this case, can make management predictions under

varied conditions of operation. The model presented is not presently suitable for this purpose, however, with some additional improvements and verification, it is expected that it can be used for such predictions.

DESCRIPTION OF THE MIAMI RIVER GROUND WATER BASIN

Figure 1 shows the surface drainage area of the Miami River basin and its general location in southwestern Ohio.

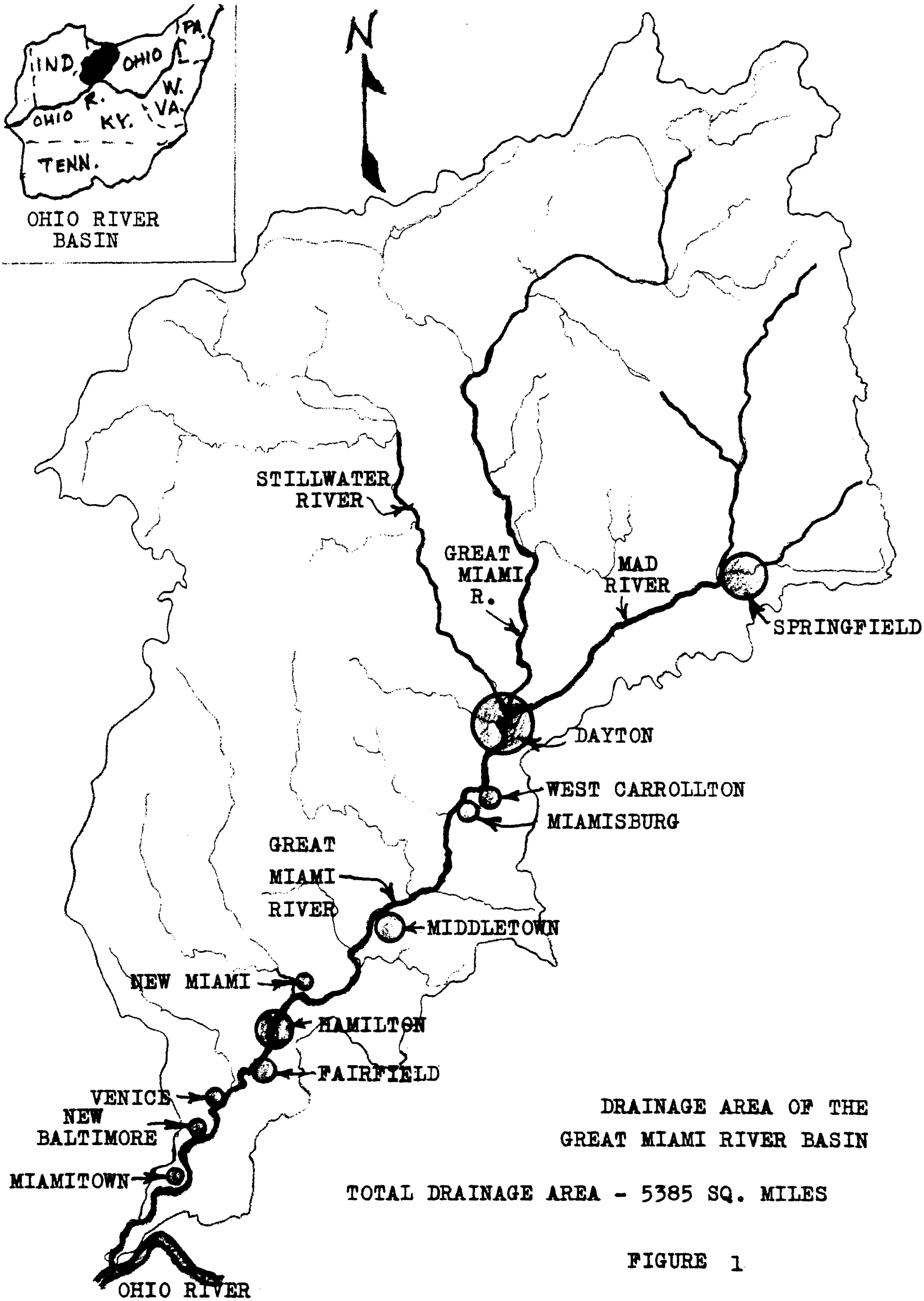
Figure 2 is a general map of the ground water resources of the Miami River basin. It can be generally noted from this Figure that the main ground water deposits follow the river closely and represent a long and narrow configuration.

The valley of the Great Miami River extending from Dayton to the Ohio River is one of the most prolific sources of ground water in midwestern United States. The major valley averages approximately two miles in width and 150' to 200' in depth and was geologically formed during the interglacial intervals of the pleistocene epoch. Subsequently, the valley was filled with highly permeable sand and gravel outwash which follows essentially the course of the Great Miami River.

For purposes of analysis, the valley has been divided into 11 hydrogeological environments on the basis of:

1. the nature of the thickness of the aquifer materials;
2. the availability of recharge by induced infiltration; and
3. the presence or absence of clay semi-confining layers.

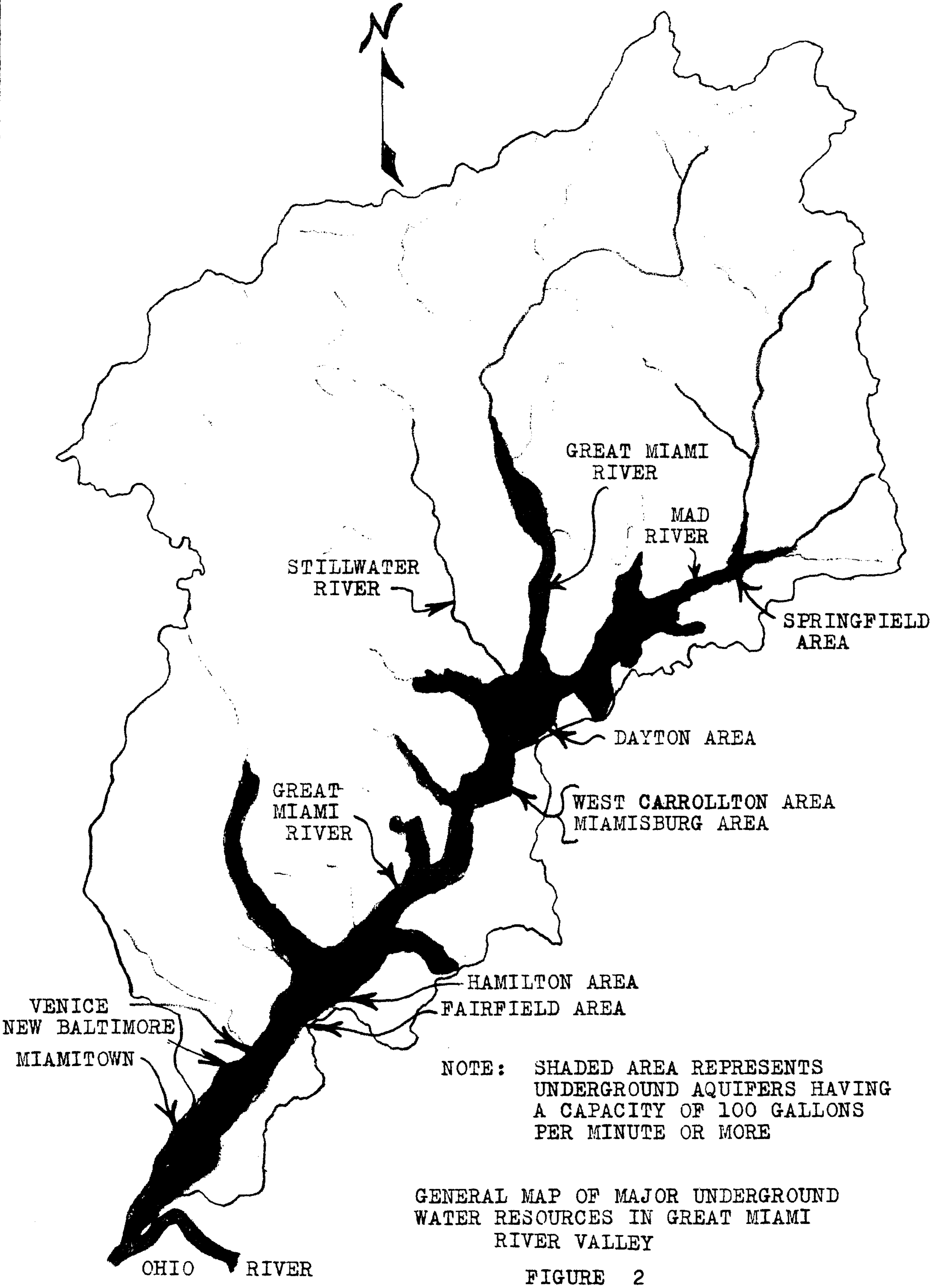
The most favorable areas for the development of large ground water supplies are in those environments where 150 feet or more sand and gravel, with no clay, are situated close enough to a major stream to permit recharge by induced stream infiltration. The most prominent of such areas are in



DRAINAGE AREA OF THE
GREAT MIAMI RIVER BASIN

TOTAL DRAINAGE AREA - 5385 SQ. MILES

FIGURE 1



GENERAL MAP OF MAJOR UNDERGROUND WATER RESOURCES IN GREAT MIAMI RIVER VALLEY

FIGURE 2

the middle and lower portions of the basin. Here the individual walls can yield as much as 3000 gpm. Only slightly less favorable are areas similarly situated with respect to streams where the aquifer is less than 150' thick or where the aquifer contains extensive layers of clay. Most of the valley north of Middletown is in this category.

In parts of the valley where the aquifer is too far from a major stream for induced infiltration or is overlain by a semi-confining clay layer, individual wells can still be expected to yield 500 gpm. Yields as high as 1000 gpm are not uncommon. Such environments are found in abandoned segments of the ancestral Great Miami Valley between West Carrollton and Carlisle, between Trenton and New Miami, and between Ross and Harrison. Smaller areas comprising this environment exist southeast of Hamilton and southeast of Middletown. The least favorable hydrogeologic environments are in tributary buried valleys filled largely or entirely with clay and in upland areas where shale bedrock is overlain by relatively impermeable glacial bed. Large groundwater supplies generally cannot be developed in these last two environments.

The discharge of the Great Miami River at Hamilton equals or exceeds 490 cfs 90% of the time. The base water is available for recharge to the aquifer by induced infiltration in warm weather under conditions of low stream flow has been determined to be in the order of 400,000 gallons per day per acre of streambed with considerably higher rates under conditions of greater streamflow. Pumpage of water which is mostly concentrated around the area's larger cities, totaled 110 mgd in 1964. Yet the ground water resources of much of the area remain untapped.

The basin gradient of the underground water surface generally follows that of the Great Miami River and is estimated at 5 to 10 feet per mile. Small cones of depression, of course, can be expected to form in areas of high usage.

The ground water levels in most of the valley are about 30' to 50' beneath the land surface and fluctuate seasonally. In winter and spring, the groundwater levels rise, and in summer and autumn, they decline. The amplitude of fluctuation ranges from 5' to 15' and is greatest in areas where the aquifer is semi-confined. The only area of chronic overdraft of the aquifer as indicated by a downward trend of the water level, is in the vicinity of Armco East Works near Middletown. Here the water level was 132' below the land surface at the end of 1966.

AREA OF STUDY

For the purpose of this study, only those aquifers which are believed to yield 100 or more gallons per minute have been included. Those with a potential yield of less than 100 gpm have been neglected. It is assumed that because of the negligible amount of water contained in them, they will not have any significant influence on the groundwater levels. The area of study is therefore assumed to be surrounded by impermeable strata and is generally shown in Figure 2.

The geometry of the aquifer is immensely complicated. However, the aquifers merge frequently to permit large transfers of water. It is assumed that the water is transferred at the boundary of the aquifer.

As shown in the more detailed Figures 4, 5, 6, 7, and 8, the basin has been divided into 26 zones. The zones were established on the basis of the aquifer characteristics and with regard to areas of general ground water level variation.

Geologic and hydrologic information as provided in reports by Spieker (1) of the U. S. Geological Survey and by Walker (2) of the State of Ohio Division of Water has been largely used for establishing the 26 aquifer zones.

For example, the zone marked No. 14 is comparatively large because of heavy pumping from this area. The aquifer characteristics are essentially the same for the entire zone.

For study purposes, hydrogeologic classifications have been assigned to the different aquifers in the basin. A hydrogeologic environment is defined as a mappable area which has underlying aquifer materials possessing distinct hydrogeologic and geologic properties differing significantly from the properties of the aquifer in adjacent areas. It is considered that ground water occurs under essentially uniform hydrologic and geologic conditions within any given hydrogeologic environment.

BASIC THEORY OF MATHEMATICAL MODEL

As an approach to the development of a mathematical model representative of the ground water dynamics of the basin, a method suggested by Tyson and Weber (3) was referred to. However, their studies dealt with the ground water basin of the Los Angeles Coastal Plain which is a rather compactly shaped area as compared with the unusually long and narrow basin under consideration in this study of the Miami River basin.

The basic theory of the approach is given below. For further details, the reader is referred to the paper by Tyson and Weber (3).

The basic equation of unsteady flow is: (Inflow) - (outflow) = (Rate of change of storage within given control volume).

Expressed mathematically, for an unconfined aquifer, in which there is no vertical variation of properties, the following equations result:

$$\nabla \bar{q} - S \frac{\partial H}{\partial t} - Q = 0$$

$$\nabla T \nabla H - S \frac{\partial H}{\partial t} - Q = 0$$

where: ∇q = inflow per unit time per unit area

Q = outflow per unit area per unit time

$S \frac{\partial H}{\partial t}$ = rate of storage per unit area

S = storage coefficient

T = transmissibility

The inflow or the replenishment flows are precipitation, imported water, stream percolation, artificial recharge, and sub-surface inflows across boundaries. The outflow or extraction flows consist mainly of the water pumped from the area for municipal water supply, industrial use, other uses, sub-surface outflow, and the discharge (efflux) of ground water into the river at certain reaches across the boundaries. For flows between aquifers Darcy's Law is used. According to Darcy's Law: $V = Ki$ for steady laminar flow.

Laushey and Popat (4) found that this is not strictly true for unsteady flow. However, the equation for unsteady flow is highly non-linear and the refinement obtained, using the unsteady Darcy's Law, is not warranted considering that the available data is itself rather approximate. Therefore, we shall use the Darcy's Law assuming a steady state and laminar flow.

Application of Continuity Equation to a Ground Water Basin:

Step 1. The ground water basin is first divided into a number of zones depending on the hydrogeologic characteristics of the aquifer. Each of these zones is assumed to be homogeneous in its properties and is a composite of the several aquifers which make up the actual structure.

The dynamic response of that portion of the model included within each zone is represented by a single water level elevation at a node within the zone at its most representative point.

The size of the zones is dependent on the variation in replenishment, extraction, transmission, storage, and water level data. It is particularly important that these zones be small in regions of large spatial rates of change of water level elevation. For the purpose of testing the model against historical data, provision is made for the extraction or injection of time varying flow rates from each of the zones.

Step 2. It is to the node points that the continuity equation is applied. The equation of continuity is replaced by an equivalent system of difference differential equations, the simultaneous solution of which gives the function H' after a time interval ∇t, at a finite number of nodes within the boundaries of the aquifer.

This can be illustrated by referring to a hypothetical basin divided into three zones with nodes i, j, k, as shown in Figure 3.

Consider node i: There is inflow from j and k. Outflow is in the form of pumping.

Inflow from j:

$$(\text{Inflow})_{ij} = (\text{Velocity})_{ij} \times (\text{Area of cross section of flow})$$

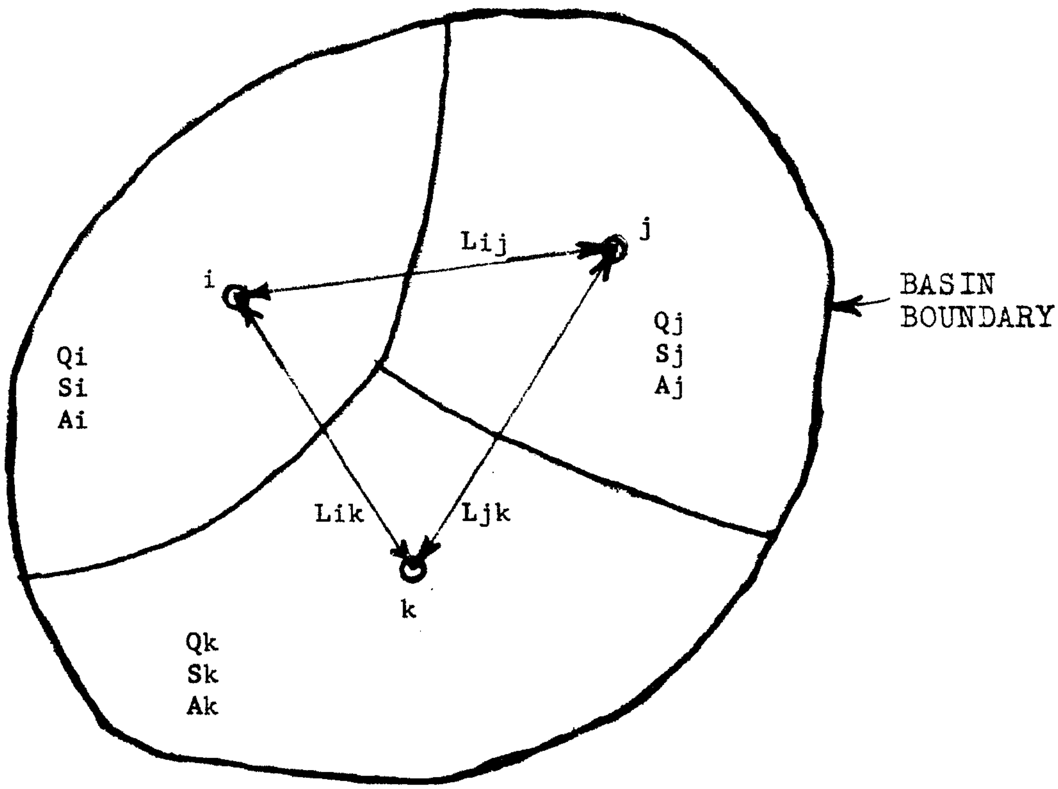


FIGURE 3

SKETCH OF HYPOTHETICAL BASIN FOR ILLUSTRATION
OF BASIC THEORY OF MATHEMATICAL MODEL

According to Darcy's Law:

$$\text{Velocity} = - K \frac{\delta H}{\delta S}$$

K = permeability of the medium

$\frac{\delta H}{\delta S}$ = hydraulic gradient

$$\text{Area of flow} = J_{ij} \cdot Y_{ij}$$

Y_{ij} = Thickness of saturated aquifer at typical cross section of flow between nodes i and j

J_{ij} = Length perpendicular to flow path between nodes i and j

$$\therefore \text{Inflow from j to i} = K \left(\frac{\delta H}{\delta S}\right)_{ij} J_{ij} Y_{ij} = T_{ij} J_{ij} \left(\frac{\delta H}{\delta S}\right)_{ij}$$

$$\left(\frac{\delta H}{\delta S}\right)_{ij} \simeq \frac{H_j' - H_i'}{L_{ij}}$$

$$\therefore \text{Inflow} = T_{ij} J_{ij} \left(\frac{H_j' - H_i'}{L_{ij}}\right) = Y_{ij} (H_j' - H_i')$$

where: T_{ij} = Length of flow path between i, j.

L_{ij} = Length of flow path between i, j.

H_i' = Head at node i after a time interval Δt.

$Y_{ij} = T_{ij} J_{ij}$ = Conductance of flow between i and j.

Inflow from k: In a similar manner inflow from node k is found as: $(\text{Inflow})_{ik} = Y_{ik} (H_k' - H_i')$

$$\therefore \text{Total inflow at i} = Y_{ij} (H_j' - H_i') + Y_{ik} (H_k' - H_i')$$

Outflow: If Q_i is the rate of pumping per unit area associated with node i and A_i is the area associated with node i, the total outflow per unit time is as follows:

$$(\text{outflow})_i = A_i \, Q_i$$

Similarly

$$(\text{outflow})_j = A_j \, Q_j$$

$$(\text{outflow})_k = A_k \, Q_k$$

Rate of change of storage:

S = storage coefficient associated with node i.

$\triangle t$ = the time interval after which the water level elevation is desired.

$(H_i' - H_i)$ = the difference in water levels after the time interval $\triangle t$ at node i.

$\therefore$ Rate of change of storage $= \frac{Si}{\triangle t} (H_i' - H_i)$

The continuity equation states that Inflow - Outflow = Rate of change of storage.

For i :

$$Y_{ij} (H_j' - H_i') + Y_{ik} (H_k' - H_i') - A_i \, Q_i = \frac{Si}{\triangle t} (H_i' - H_i) \qquad \text{--1}$$

Similarly for k ;

$$Y_{ki} (H_i' - H_k') + Y_{kj} (H_j' - H_k') - A_k \, Q_k = \frac{Sk}{\triangle t} (H_k' - H_k) \qquad \text{--2}$$

For j ;

$$Y_{ji} (H_i' - H_j') + Y_{jk} (H_k' - H_j') - A_j \, Q_j = \frac{Sj}{\triangle t} (H_j' - H_j) \qquad \text{--3}$$

Solving the above three equations simultaneously, the water levels H_i', H_j', H_k' after a time interval Δt are found.

The continuity equation is applied to the Miami River Basin in the manner illustrated above.

APPLICATION OF MATHEMATICAL MODEL TO THE GROUND WATER BASIN OF THE GREAT MIAMI RIVER

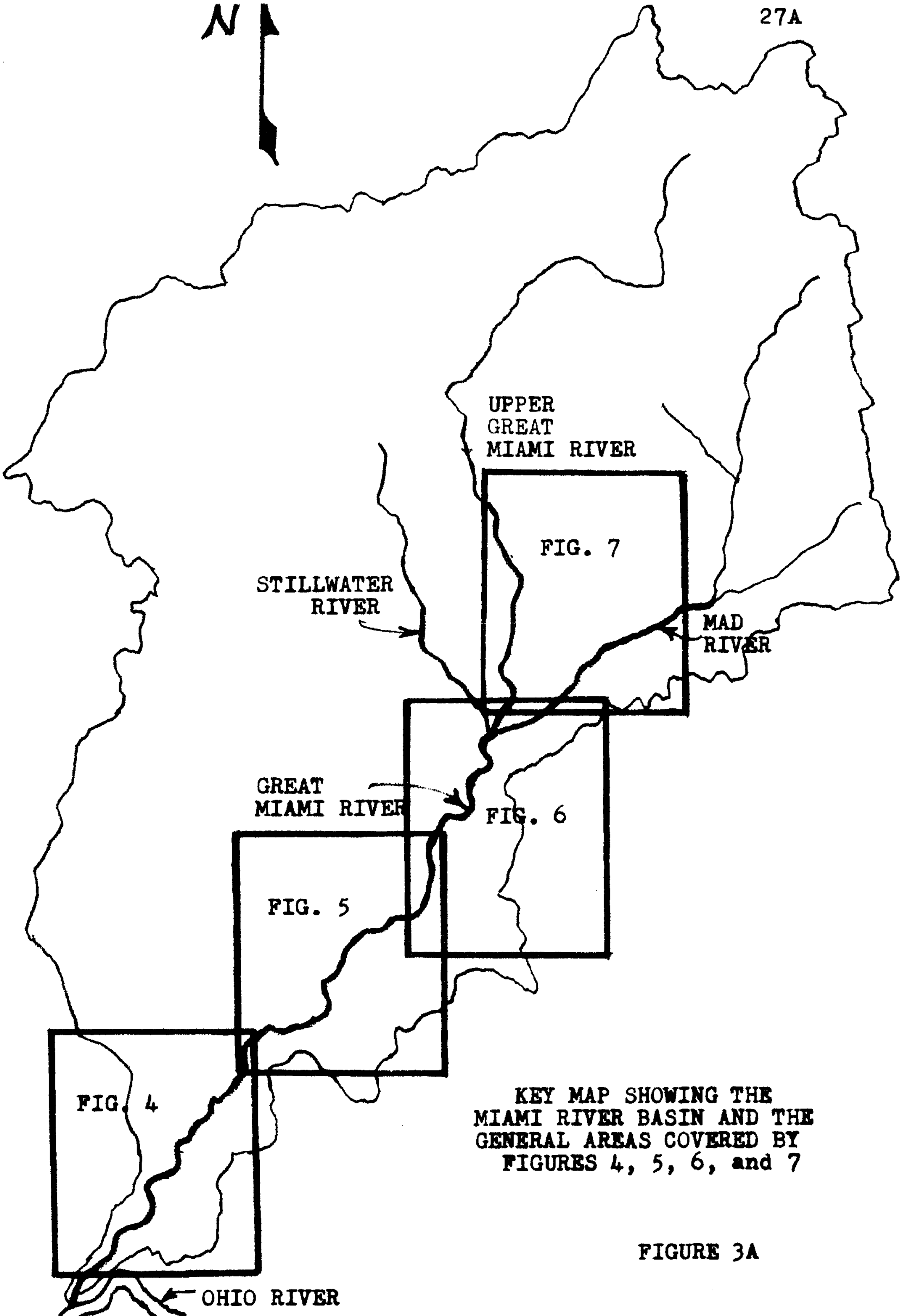
The 26 zoned areas shown in Figures 4, 5, 6, and 7 are represented by 26 node points located approximately at the centroids of the areas. The node points are used to indicate ground water elevation changes in the respective zones.

In order to establish the hydraulic characteristics of the aquifers in the zones, a classification system has been established as shown in Table 1. The aquifers have been divided into nine groupings indicating their general degree of favorability for ground water development. The system shown in Table 1 is based on the following criteria as suggested by Spieker (1):

1. The hydraulic and geologic nature of the aquifer;
2. The availability of recharge by induced infiltration;
3. presence or absence of interstratified clay layers; and
4. thickness of the aquifer.

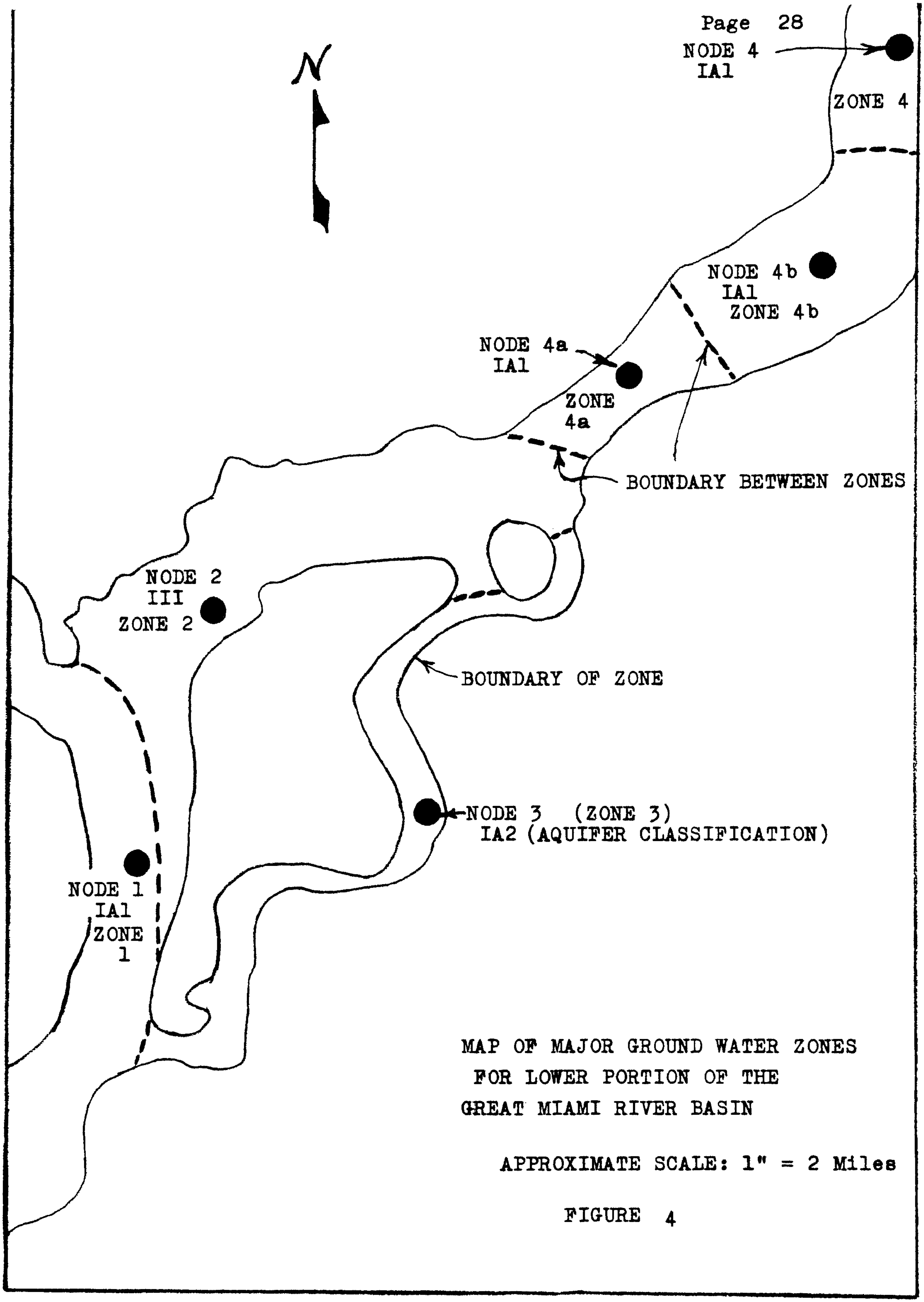
It will be noted that the classifications in Table 1 are listed from top to bottom in order of their decreasing favorability with respect to their potential for ground water development. In Figures 4, 5, 6, and 7, the classifications for respective zones have been shown.

The most favorable environments for the development of large ground water supplies in the Great Miami River valley are in those areas where 150' or more of sand and gravel are available. The classification designated I A 1 is present in three areas of the valley: the area in the vicinity of Trenton; the area immediately southwest of Middletown; and the area extending from north of New Miami through Hamilton and



KEY MAP SHOWING THE MIAMI RIVER BASIN AND THE GENERAL AREAS COVERED BY FIGURES 4, 5, 6, and 7

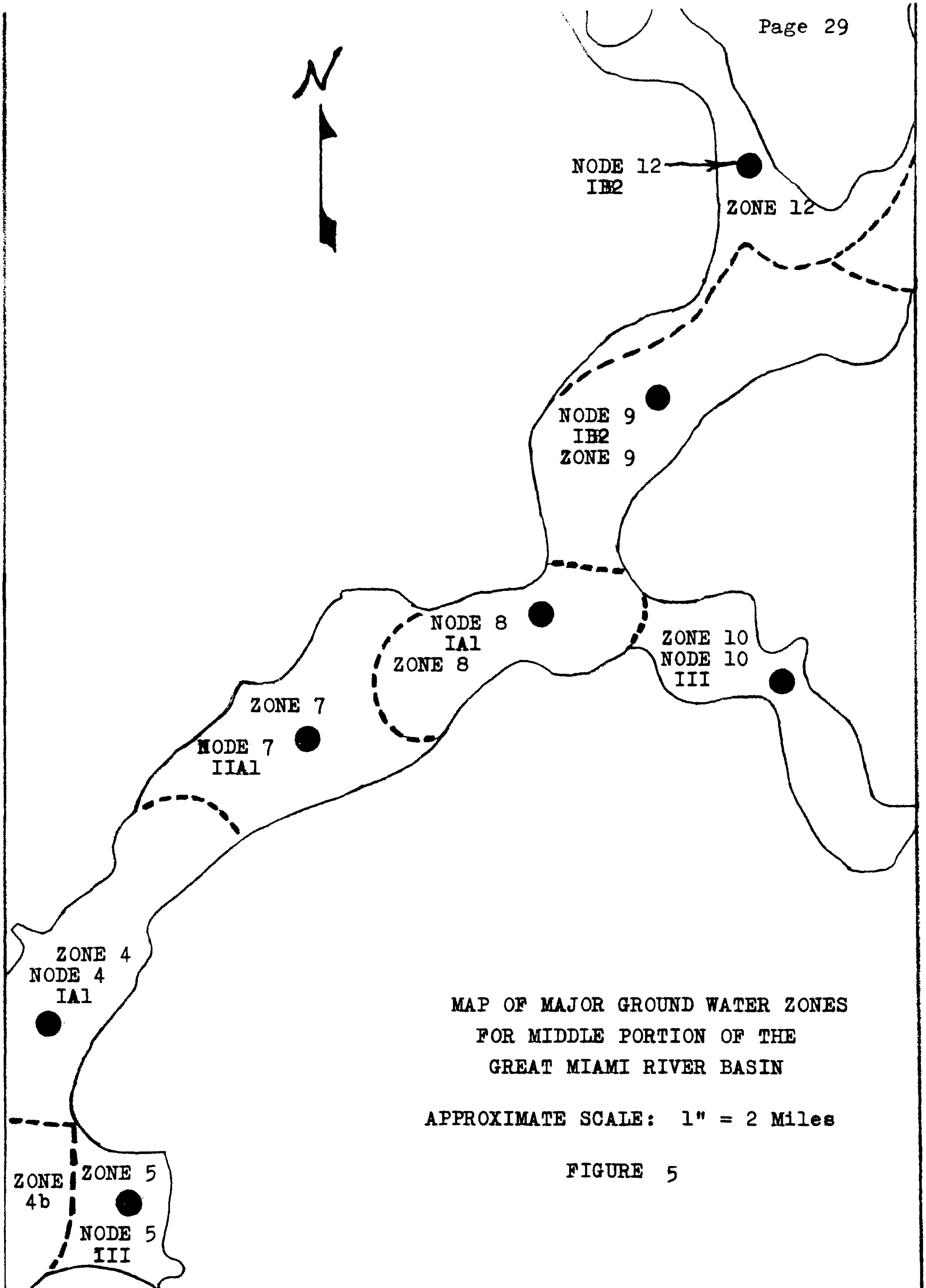
FIGURE 3A



MAP OF MAJOR GROUND WATER ZONES
FOR LOWER PORTION OF THE
GREAT MIAMI RIVER BASIN

APPROXIMATE SCALE: 1" = 2 Miles

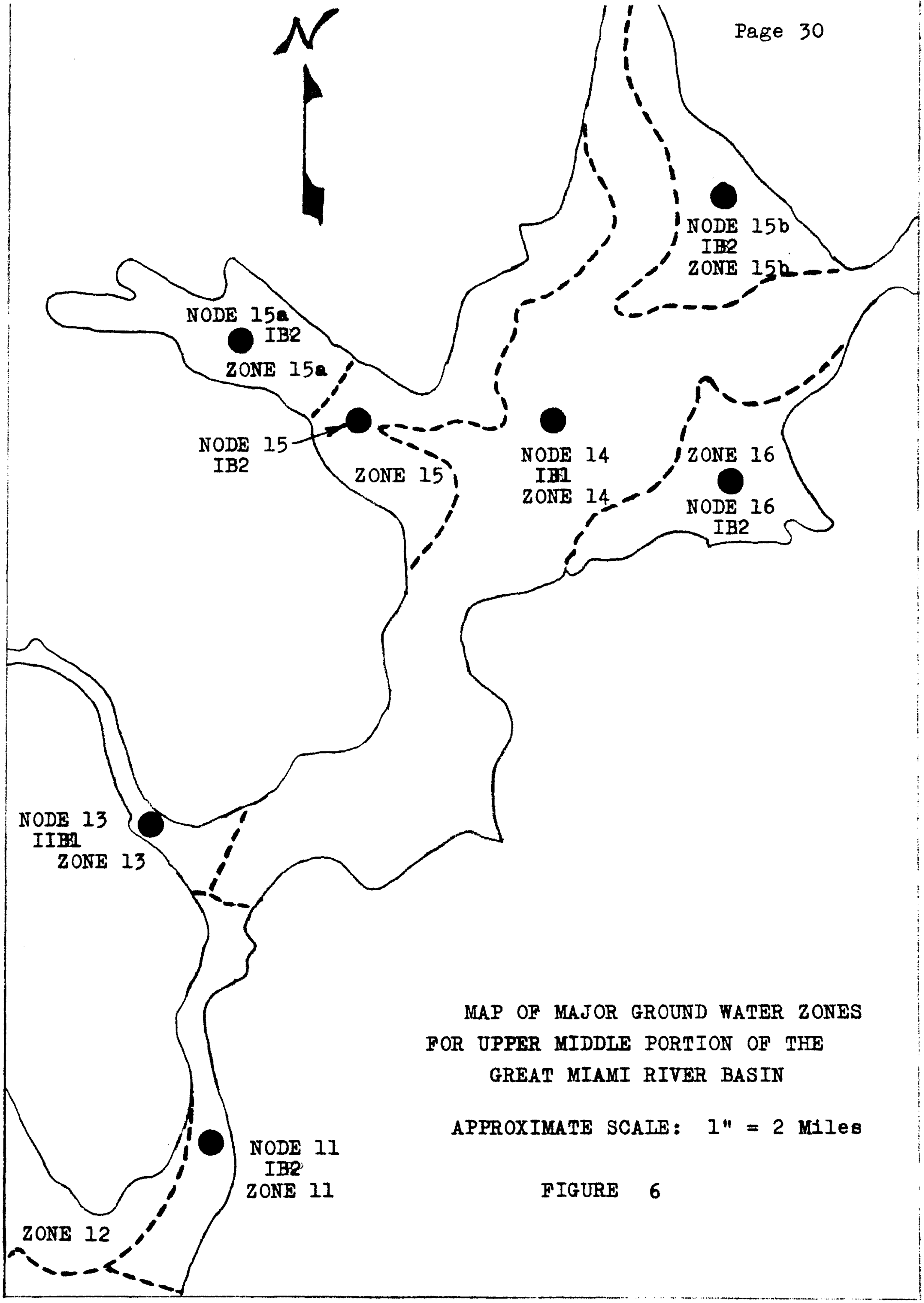
FIGURE 4



MAP OF MAJOR GROUND WATER ZONES
FOR MIDDLE PORTION OF THE
GREAT MIAMI RIVER BASIN

APPROXIMATE SCALE: 1" = 2 Miles

FIGURE 5



MAP OF MAJOR GROUND WATER ZONES
FOR UPPER MIDDLE PORTION OF THE
GREAT MIAMI RIVER BASIN

APPROXIMATE SCALE: 1" = 2 Miles

FIGURE 6

MAP OF MAJOR GROUND WATER ZONES
FOR UPPER MIDDLE PORTION OF THE
GREAT MIAMI RIVER BASIN
AND LOWER MAD RIVER BASIN

APPROXIMATE SCALE: 1" = 2 Miles

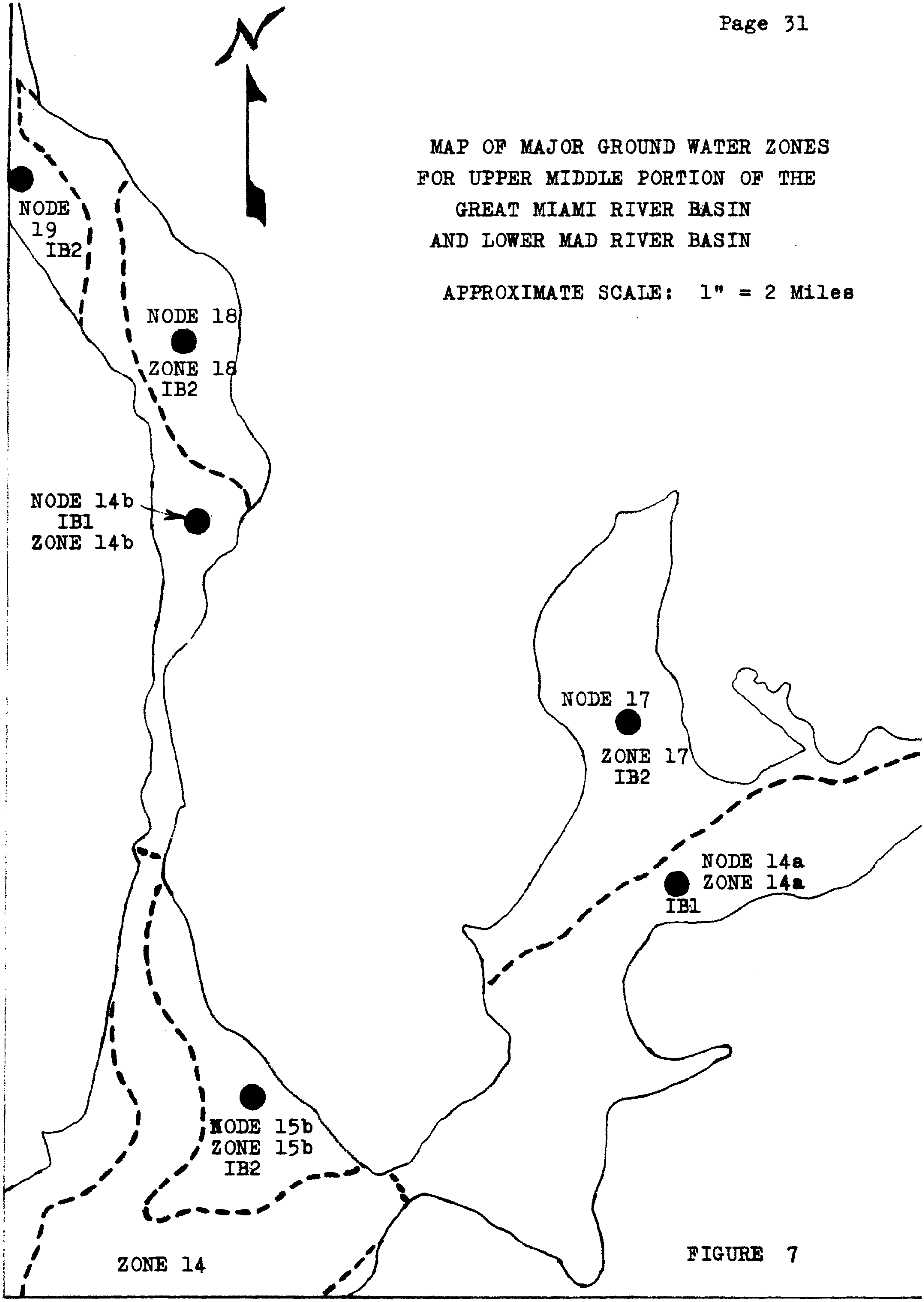


FIGURE 7

TABLE 1

CLASSIFICATIONS OF AQUIFERS IN GREAT MIAMI RIVER VALLEY

Classification	Aquifer Description
IA1	Sand and gravel Recharge by induced stream infiltration available No interstratified clay layers present Thickness, 150 - 200 ft or more
IA2	Sand and gravel Recharge by induced stream infiltration available No interstratified clay layers present Thickness, less than 150 ft
IB1	Sand and gravel Recharge by induced stream infiltration available Interstratified clay layers may be present Thickness, 150 - 200 ft or more
IB2	Sand and gravel Recharge by induced stream infiltration available Interstratified clay layers may be present Thickness, less than 150 ft
IIA1	Sand and gravel No recharge by induced stream infiltration available No interstratified clay layers present Thickness, 150 - 200 ft or more
IIA2	Sand and gravel No recharge by induced stream infiltration available No interstratified clay layers present Thickness, less than 150 ft
IIB1	Sand and gravel No recharge by induced stream infiltration available Interstratified clay layers may be present Thickness, 150 - 200 ft or more
IIB2	Sand and gravel No recharge by induced stream infiltration available Interstratified clay layers may be present Thickness, less than 150 ft
III	Sand and gravel overlain by clay Stream recharge generally not available

Fairfield to west of Ross and the lower Whitewater River valley southeast of Harrison. Several of the largest groundwater supplies in the lower Great Miami River valley are located in this environment.

The coefficient of transmissibility of the aquifer in classification IA1 ranges generally from 300,000 to 500,000 gpd per ft. The coefficient of storage (S) is about 0.2. Properly constructed individual wells can yield as much as 3000 gpm or more, and specific capacities are in the range of 3000 gpm per foot of drawdown.

Information similar to the above has been established for all of the aquifer classifications.

For the initial computer analysis, the ground water complex is replaced by a simplified model. This model consists of a single equivalent aquifer whose local properties are composites of the corresponding properties of the several aquifers. The thickness of the single aquifer is allowed to vary with position, and yet it is considered to be small compared to its lateral dimensions.

The dynamic response of the portion of the model included within each zone is represented by a single water level evaluation. The size of a zone is dependent on the variation in replenishment, extraction, transmission, storage, and water level data. It is particularly important that these zones be small in regions of large spatial rates of change of water level elevation. For the purpose of testing the model against historical data, provision has been made for the extraction or injection of time-varying flow rates from each of the zones.

The approximation is made that the water level or the dynamic response of the entire zone is represented by a node point within the zone at its centroid or its most representative point. The equation or continuity is replaced by an equivalent system of difference differential equations, the simultaneous solution of which gives the water surface elevation, H', for a constant rate of pumpage and inflow during a time interval Δt, at a finite number of node points within the boundaries of the aquifer.

Knowing the precipitation, induced infiltration, and the pumpage, it is thus possible to predict the level of the water table in these zones after a time interval Δt.

However, a correction must be made for the time taken for a particle of water to arrive at the node towards which flow occurs. To estimate the arrival time of the water particle, the method adopted by Nelson and Eliason (5) in their paper "Prediction of Water Movement Through Soils - A First Step in Waste Transport Analysis" is referred to.

The time of travel is found by evaluating the integral of the reciprocal of the velocity along the length of the flow path involved.

$$T = \int_{1}^{2} \frac{dL}{V} = \int_{x_1}^{x_2} \frac{dx}{Vx} = \int_{y_1}^{y_2} \frac{dy}{Vy} = \int_{z_1}^{z_2} \frac{dz}{Vz}$$

here:

L = length of flow path

$$Tm = \frac{P}{K} \sum_{I}^{N-1} \left(\frac{\lambda^2}{\Delta \varphi}\right) i$$

here:

Tm - mean travel time along flow path

P - porosity

K - hydraulic conductivity of soil

λ - distance measured between equipotentials along the streamline of interest

$\Delta\phi$ - difference in potential (length units) between equipotentials which are a distance apart

N - number of equipotentials along stream line

The above method was developed for the cases where flow occurs from a stream to an aquifer or where flow occurs between aquifers. Where pumping occurs from the aquifers, it is considered in this report that the same approach can be used with a variation in the determination of the velocity between two adjoining zones. The method for calculation of velocity used for the model in this report is described below.

Flow between the zones is considered to occur at their mutual boundaries as shown in Figure 8. The cross-sectional shape of the boundary is assumed to be rectangular. Heights of the rectangular boundaries are averaged from the contours of the bed rock and ground water levels. Figure 8 shows to adjoining zones with pumpages, Q_1 and Q_2, occurring from the respective nodes. The velocity of flow between nodes 1 and 2 is calculated from:

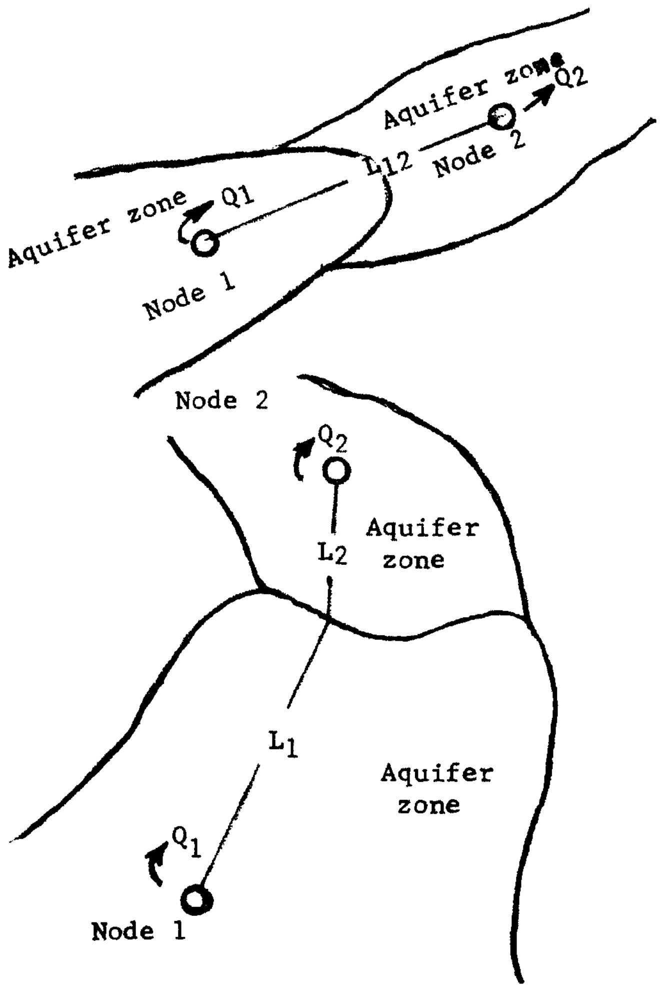
$$V_{12} = \frac{Q_1 - Q_2}{A_{12}}$$

where: A_{12} is the boundary area of flow between zones 1 and 2.

The time of travel between nodes 1 and 2 is then calculated from:

$$T_{12} = \frac{L_{12}}{V_{12}}$$

where: L_{12} is the distance between nodes as indicated in Figure 8.



CASE I

CASE II $L_1 + L_2 = L_{12}$

Q_1 - Pumpage at 1
Q_2 - Pumpage at 2
A_{12} - Flow area between 1 and 2
L_{12} - Distance between nodes
T_{12} - Time of travel between 1 and 2

FIGURE 8

SKETCH ILLUSTRATING FLOW BETWEEN AQUIFER ZONES

NOTATION FOR MATHEMATICAL COMPUTER MODEL

L_{12}	Length of the path of flow between nodes 1 and 2. (feet)
AJ_{12}	Length of the boundary perpendicular to the direction of flow.
T_1	Transmissibility associated with zone 1. gpd/ft
T_{12}	Equivalent transmissibility associated with the flow between 1 and 2. gpd/ft
P	Permeability
DIS_1	Extraction associated with node 1. (mgd.)
R_4	Precipitation (inches)
$INFILT_1$	Inflow due to induced infiltration (cft/s)
S_1	Storage coefficient
A_1	Area sq. ft.
DH_{12}	Difference in heads between two nodes (feet)
D_{12}	Average depth of the aquifer at the boundary of the nodes
Y_{12}	Conductance between two nodes
H_1'	Head at the end of time interval ft
H_1	Head at the beginning of the time interval ft

The subscripts denote nodes with which the symbols are associated.

<u>Node 1</u>:

<u>Inflow</u> Inflow from 2 and 3.

$$\text{Inflow} = \sum_{2,3} KAi = K_{12}\ J_{12}\ D_{12}\ \frac{H_2' - H'}{L_{12}} + K_{13}\ J_{13}\ D_{13} \times \frac{H_3' - H_1'}{L_{13}}$$

$$= Y_{21}\ (H_2' - H_1' - H_1') + Y_{31}\ (H_3' - H_1')$$

Precipitation = $R_1 \times A_1/12$

Infiltration: from Stillwater River $INFILT_1$

Outflow

Pumpage = $DIS_1 \times \triangle T$

Storage Rate

$S_1 A_1 (H_1' - H_1)/\triangle T$

$\therefore$ Continuity equation for Node 1 is:

$$Y_{21} (H_2' - H_1') + Y_{31} (H_3' - H_1') + R_1 A_{1/12} + INFILT_1 - DIS_1 \times \triangle T =$$

$$\frac{S_1 A_1}{\triangle T} (H_1' - H_1)$$

Node 2:

Inflow from 1, 3, and 4a

$$\text{Inflow} = \sum_{1,3,4a} KAi = Y_{21} (H_1' - H_2') + Y_{23} (H_3' - H_2') + Y_{24a} (H_{4a}' - H_2')$$

Precipitation: $R_2 A_{2/12}$

Infiltration: $INFILT_2$

Outflow $DIS_2 \times \triangle T$

$\therefore$ Continuity equation yields

$$Y_{21} (H_1' - H_2') + Y_{23} (H_3' - H_2') + Y_{24a} (H_{4a}' - H_2') + R_2 A_{2/12} + INFILT_2$$

$$- DIS_2 \triangle T = S_2 A_{2/\triangle T} (H_2' - H_2)$$

Node 3: Inflow from 1, 2.

Inflow

$$\text{Inflow} = \sum_{1,2} KAi = Y_{31} (H_1 - H_3') + Y_{32} (H_2' - H_3')$$

Precipitation: $R_3\ A_{3/12}$

Infiltration = $INFILT_3$

Outflow $DIS_3 \Delta T$

Continuity equation:

$$Y_{31}\ (H_1' - H_3') + Y_{32}\ (H_2' - H_3') + R_3\ A_{3/12} + INFILT_3 - DIS_3 \Delta T =$$

$$\frac{S_3\ A_3}{\Delta T}\ (H_3'\ H_3)$$

Node 4a: Inflow from 4b, 2

Continuity equation:

$$Y_{4a4b}\ (H_{4b}' - H_{4a}') + Y_{4a2}\ (H_2' - H_{4a}') + R_{4a}\ A_{4a/12} + INFILT_{4a} - DIS_{4a} \Delta T$$

$$= \frac{S_{4a}A_{4a}}{\Delta T}\ (H_{4a}' - H_{4a})$$

Node 4b: Inflow from 4, 5, 4a

Continuity equation:

$$Y_{4b4}\ (H_4' - H_{4b}') + Y_{4b5}\ (H_5' - H_{4b}') + Y_{4b4a}\ (H_{4a}' - H_{4b}') + R_{4b} \times A_{4b/12}$$

$$+ INFILT_{4b} - DIS_{4b} \Delta T = \frac{S_{4b}A_{4b}}{\Delta T}\ (H_{4b}' - H_{4b})$$

Node 4: Inflow from 4b, 6, 7

Continuity equation:

$$Y_{44b}\ (H'_{4b} - H'_4) + Y_{4b6}\ (H'_6 - H'_4) + Y_{47}\ (H'_9 - H'_4) + R_4\ A_{4/12} +$$

$$INFILT_4 - DIS_4 \triangle T = \frac{S_4 A_4}{\triangle T}\ (H'_4 - H_4)$$

<u>Node 5</u>: Inflow from 4b

Continuity equation:

$$Y_{54b}\ (H'_{4b} - H'_5) + R_5\ A_{5/12} + INFILT_5 - DIS_4 \triangle T = \frac{S_4 A_4}{\triangle T}(H'_5 - H_5)$$

<u>Node 6</u>: Inflow from 6a, 4, 7

Continuity equation:

$$Y_{66a}\ (H'_{6a} - H'_6) + Y_{64}\ (H'_4 - H'_6) + Y_{67}\ (H'_7 - H'_6) + R_6\ A_{6/12} + INFILT_6$$

$$- DIS_6 \triangle T = \frac{S_6 A_6}{\triangle T}\ (H'_6 - H_6)$$

<u>Node 7</u>: Inflow from 6, 4, 8

Continuity equation:

$$Y_{76}\ (H'_6 - H'_7) + Y_{74}\ (H'_4 - H'_7) + Y_{78}\ (H'_8 - H'_7) + R_7\ A_{7/12} + INFILT_7$$

$$- DIS_7 \triangle T = \frac{S_7 A_7}{\triangle T}\ (H'_7 - H_7)$$

<u>Node 6a</u>: Inflow from 6

Continuity equation:

$$Y_{6a6}\ (H'_6 - H'_{6a}) + R_{6a}\ A_{6a/12} + INFILT_{6a} - DIS_{6a} \triangle T = \frac{S_{6a} A_{6a}}{\triangle T}\ (H'_{6a} - H'_{6a})$$

<u>Node 8</u>: Inflow from 7, 10, 9

Continuity equation:

$$Y_{87}\ (H'_7 - H'_8) + Y_{810}\ (H'_{10} - H'_8) + Y_{89}\ (H'_9 - H'_8) + R_8\ A_{8/12} + INFILT_8$$

$$- DIS_8 \triangle T = \frac{S_8 A_8}{\triangle T}\ (H'_8 - H_8)$$

<u>Node 9</u>: Inflow from 12, 11, 8

Continuity equation:

$$Y_{912}\ (H'_{12} - H'_9) + Y_{911}\ (H'_{11} - H'_9) + Y_{98}\ (H'_8 - H'_9) + R_9\ A_{9/12} + INFILT_9$$

$$- DIS_9 \triangle T = \frac{S_9 A_9}{\triangle T}\ (H'_9 - H_9)$$

<u>Node 10</u>: Inflow from 8

Continuity equation:

$$Y_{810}\ (H'_8 - H'_{10}) + INFILT_{10} + R_{10}\ A_{10/12} - DIS_{10} \triangle T = \frac{S_{10} A_{10}}{\triangle T}\ (H'_{10} - H_{10})$$

<u>Node 11</u>: Inflow from 12, 9, 14

Continuity equation:

$$Y_{1112}\ (H'_{12} - H'_{11}) + Y_{119}\ (H'_9 - H'_{11}) + Y_{1114}\ (H'_{14} - H'_{11}) + INFILT_{11}$$

$$+ R_{11}\ A_{11/12} - DIS_{11} \triangle T = \frac{S_{11} A_{11}}{\triangle T}\ (H'_{11} - H_{11})$$

<u>Node 12</u>: Inflow from 9, 11

Continuity equation:

$$Y_{129}\ (H'_9 - H'_{12}) + Y_{1211}\ (H'_{11} - H'_{12}) + INFILT_{12} + R_{12}\ A_{12/12} - DIS_{12} \triangle T$$

$$= \frac{S_{12} A_{12}}{\triangle T}\ (H'_{12} - H_{12})$$

Node 13: Inflow from 14

Continuity equation:

$$Y_{1314}\ (H'_{14} - H'_{13}) + INFILT_{14} + R_{14}\ A_{14/12} - DIS_{14} \triangle T = \frac{S_{13}A_{13}}{\triangle T}\ (H'_{13} - H_{13})$$

Node 14: Inflow from 13, 11, 15, 15b, 14b, 14c, 16

Continuity equation:

$$Y_{1413}\ (H'_{13} - H'_{14}) + Y_{1411}\ (H'_{11} - H'_{14}) + Y_{1514}\ (H'_{15} - H'_{14}) + Y_{1415b}$$

$$(H'_{15b} - H'_{14}) + Y_{1414b}\ (H'_{14b} - H'_{14}) + Y_{1414c}\ (H'_{14c} - H'_{14}) + INFILT_{14}$$

$$+ R_{14}\ A_{14/12} + Y_{1416}\ (H'_{16} - H_{14}) - DIS_{14} \triangle T = \frac{S_{14}A_{14}}{\triangle T}\ (H'_{14} - H_{14})$$

Node 14b: Inflow from 18, 19, 14

Continuity equation:

$$Y_{14b18}\ (H'_{18} - H'_{14b}) + Y_{14b19}\ (H'_{19} - H'_{14b}) + Y_{14b14}\ (H'_{14} - H'_{14b})$$

$$+ INFILT_{14b} + R_{14b}A\ _{14b/12} - DIS_{14b} \triangle T = \frac{S_{14b}A_{14b}}{\triangle T}(H'_{14b} - H_{14b})$$

Node 14c: Inflow from 14, 17

Continuity equation:

$$Y_{14c14}\ (H'_{14} - H'_{14c}) + Y_{14c17}\ (H'_{17} - H'_{14c}) + INFILT_{14c} + R_{14c}\ A_{14c/12}$$

$$- DIS_{14c} \triangle T = \frac{S_{14c}A_{14c}}{\triangle T}\ (H'_{14c} - H_{14c})$$

Node 15b: Inflow from 14

Continuity equation:

$$Y_{15b14}\ (H'_{14} - H'_{15b}) + INFILT_{15b} + R_{15b}\ A_{15b/12} - DIS_{15b} \triangle T =$$

$$\frac{S_{15b}A_{15b}}{\triangle T}\ (H'_{15} - H_{15})$$

Node 15: Inflow from 15c, 14

Continuity equation:

$$Y_{1515c}\,(H'_{15c} - H'_{15}) + Y_{1514}\,(H'_{14} - H'_{15}) + INFILT_{15} + R_{15}\,A_{15/12}$$

$$- DIS_{15}\,\Delta T = \frac{S_{15}A_{15}}{\Delta T}\,(H'_{15} - H_{15})$$

Node 15c: Inflow from 15

Continuity equation:

$$Y_{15c15}\,(H'_{15} - H'_{15c}) + INFILT_{15c} + R_{15c}\,A_{15c/12} - DIS_{15c}\,\Delta T =$$

$$\frac{S_{15c}A_{15c}}{\Delta T}\,(H'_{15c} - H_{15c})$$

Node 16: Inflow from 14

Continuity equation:

$$Y_{1614}\,(H'_{14} - H'_{16}) + INFILT_{16} + R_{16}\,A_{16/12} - DIS_{14}\,\Delta T = \frac{S_{16}A_{16}}{\Delta T}\,(H'_{16} - H_{16})$$

Node 17: Inflow from 14c

Continuity equation:

$$Y_{14c17}\,(H'_{14c} - H'_{17}) + INFILT_{17} + R_{17}\,A_{17/12} - DIS_{17}\,\Delta T =$$

$$\frac{S_{17}A_{17}}{\Delta T}\,(H'_{17} - H_{17})$$

Node 18: Inflow from 14b

Continuity equation:

$$Y_{14b18}\,(H'_{14b} - H'_{18}) + INFILT_{18} + R_{18}\,A_{18/12} - DIS_{18}\,\Delta T =$$

$$\frac{S_{18}A_{18}}{\Delta T}\,(H'_{18} - H_{18})$$

Node 19: Inflow from 14b

Continuity equation:

$$Y_{14b19}\ (H'_{14b} - H'_{19}) + INFILT_{19} + R_{19}\ A_{19/12} - DIS_{19}\ \triangle T =$$

$$\frac{S_{19}A_{19}}{\triangle T}\ (H'_{19} - H_{19})$$

Simplifying the equations and putting them in the form of matrix notation, the following matrix equation results:

$$(K)\ (H') = (Q)$$

From this equation, the values of H' can be obtained after a time $\triangle T$.

H' values are obtained for time increments from 30 days up to 360 days. These values are matched with the actual values, and if they agree, the model can then be used for predicting future values at a given rate of extraction and replenishment.

The Average Method:

The above method considers the difference in heads at the end of the period in calculating the inflow. However, actually since the flow takes place throughout the time, $\triangle T$, an average of the head difference before and after the period gives a better evaluation.

The continuity equation is again applied to each of the nodes as done previously. The only change made is that the head difference is replaced by the average of the initial and final heads.

For example, applying this to Node 2, we have:

$$Y_{21}\ \frac{(H'_1 - H'_2 + H_1 - H_2)}{2} + Y_{23}\ \frac{(H'_3 - H'_2 + H_3 - H_2)}{2} + R_2\ A_{2/12} + INFILT_2$$

$$- DIS_2\ \triangle T = \frac{S_2A_2}{\triangle T}\ (H'_2 - H_2)$$

Applying this in the same manner to all the nodes and simplifying, the following matrix equation results:

$$(K)\ (H') = (K_t)\ (H) + (Q)$$

End of the Period Method:

The matrix equation is:

$$(K)\ (H') = (Q)$$

Q = Flow or extraction-Replenishment matrix.

$$(H') = (K)^{-1}\ (Q)$$

K = Conductance matrix.

H' = Heads at the end of period at each node.

The following are the elements in the K matrix:

$$K_{11} = S_1\ A_1/\triangle T + Y_{21} + Y_{31}$$

$$K_{22} = S_2\ A_2/\triangle T + Y_{12} + Y_{23} + Y_{24a}$$

$$K_{33} = S_3\ A_3/\triangle T + Y_{31} + Y_{32}$$

$$K_{44} = S_4\ A_4/\triangle T + Y_{4b4} + Y_{64} + Y_{74}$$

$$K_{55} = S_5\ A_5/\triangle T + Y_{24a} + Y_{4a4b}$$

$$K_{66} = S_6\ A_6/\triangle T + Y_{4a4b} + Y_{44b} + Y_{4b5}$$

$$K_{77} = S_7\ A_7/\triangle T + Y_{4b5}$$

$$K_{88} = S_8\ A_8/\triangle T + Y_{46} + Y_{6a6} + Y_{76}$$

$$K_{99} = S_9\ A_9/\triangle T + Y_{66a}$$

$$K_{1010} = S_{10}\ A_{10}/\triangle T + Y_{47} + Y_{67} + Y_{87}$$

$$K_{1111} = S_{11}\ A_{11}/\triangle T + Y_{78} + Y_{98} + Y_{108}$$

$$K_{1212} = S_{12}\ A_{12}/\triangle T + Y_{89} + Y_{119} + Y_{129}$$

$$K_{1313} = S_{13} A_{13}/\Delta T + Y_{810}$$

$$K_{1414} = S_{14} A_{14}/\Delta T + Y_{911} + Y_{1211} + Y_{1411}$$

$$K_{1515} = S_{15} A_{15}/\Delta T + Y_{912} + Y_{1112}$$

$$K_{1616} = S_{16} A_{16}/\Delta T + Y_{1413}$$

$$K_{1717} = S_{17} A_{17}/\Delta T + Y_{1114} + Y_{1314} + Y_{14b14} + Y_{14c14} + Y_{1514}$$

$$K_{1818} = S_{18} A_{18}/\Delta T + Y_{1414b}$$

$$K_{1919} = S_{19} A_{19}/\Delta T + Y_{1414c}$$

$$K_{2020} = S_{20} A_{20}/\Delta T + Y_{1415} + Y_{1515c}$$

$$K_{2121} = S_{21} A_{21}/\Delta T + Y_{1515c}$$

$$K_{2222} = S_{22} A_{22}/\Delta T + Y_{1415c}$$

$$K_{2323} = S_{23} A_{23}/\Delta T + Y_{1416}$$

$$K_{2424} = S_{24} A_{24}/\Delta T + Y_{1714c}$$

$$K_{2525} = S_{25} A_{25}/\Delta T + Y_{1814b}$$

$$K_{2626} = S_{26} A_{26}/\Delta T + Y_{1914b}$$

$$K_{12} = -Y_{(1)}$$

$$K_{13} = -Y_{(2)}$$

$$K_{23} = -Y_{(3)}$$

$$K_{25} = -Y_{(4)}$$

$$K_{46} = -Y_{(8)}$$

$$K_{48} = -Y_{(11)}$$

$$K_{410} = -Y_{(10)}$$

$$K_{56} = -Y_{(7)}$$

$K_{67} = -Y_{(9)}$

$K_{89} = -Y_{(30)}$

$K_{810} = -Y_{(12)}$

$K_{1011} = -Y_{(13)}$

$K_{1112} = -Y_{(14)}$

$K_{1113} = -Y_{(15)}$

$K_{1214} = -Y_{(17)}$

$K_{1415} = -Y_{(18)}$

$K_{1417} = -Y_{(19)}$

$K_{1617} = -Y_{(20)}$

$K_{1718} = -Y_{(26)}$

$K_{1719} = -Y_{(25)}$

$K_{1720} = -Y_{(21)}$

$K_{1722} = -Y_{(24)}$

$K_{1723} = -Y_{(23)}$

$K_{1825} = -Y_{(27)}$

$K_{1826} = -Y_{(28)}$

$K_{1924} = -Y_{(29)}$

'$\underline{Q}$' = A column matrix

Q_I = H (I) S (I) A (I)/$\triangle$ T - (DIS (I) - R (I) INF. A (I)/12)
+ INFILT (I)

H(I) = Head at beginning of period at Node I

S(I) = Storage coefficient at I

A(I) = Area associated with I

DIS(I) = Pumpage at Node I

R(I) = Rainfall at I

Inf = % of precipitation entering aquifer

Average Period Method:

Matrix Equation is: (K_-) (H') = (K_+) (H) + (Q)

Now multiplying both sides by -1 we have:

(K+) (H') = (K-) (H) - (Q)

where

$$K+ (I,I) = (K (I,I) + S (I)/IDT)/2.$$

$$K- (I,I) = -K (I,I)/2 + 1.5 \cdot S (I) \cdot A (I)/IDT.$$

$$I \neq J \quad K+ (I,J) = K (I,I)/2$$

$$I \neq J \quad K- (I,J) = -K (I,J)/2$$

K+ (I,J) - refers to the element of K+ matrix located by row I and column J.

K- (I,J) - refers to the element of K- matrix located by row I and column J.

K - refers to the conductance matrix 'K' used for the end of the period method.

VERFICATION OF THE MATHEMATICAL COMPUTER MODEL

Two computer programs have been developed for use of the model with and IBM 7040 computer. One of the programs has been developed for the "End of Period" method and the other for the "Average Period" method.

A large amount of data has been gathered through the Miami Conservancy District of Dayton, Ohio; the Division of Water, Department of Natural Resources, State of Ohio in Columbus; the U. S. Geological Survey in Columbus, Ohio; the Southwestern Ohio Water Company of Cincinnati, Ohio; and various local agencies. The cooperation and suggestions provided by these agencies is gratefully acknowledged.

The data obtained have been compiled and used to test the mathematical model on a preliminary basis. At this time, the results of these verification tests are not ready for publication. Preliminary results indicate that some addition field data may be required in order to properly verify the model. It is intended that information of this nature will be included in a later report dependent upon continued support of the project.

The computer program model as devised is considered to be valid for use in its present form. However, the values used for basic parameters will need to be refined as additional field data become available. This can easily be done in the program by substitution of the correct values in the computer program cards.

PROPOSED USES FOR THE MATHEMATICAL MODEL

Upon satisfactory verification of the model, the following uses are visualized:

1. It should be possible to predict ground water levels at desired points in the basin under varied conditions of inflow and outflow.

2. It should be possible to estimate the maximum allowable pumping extractions so that serious overdrafts in the basin will not occur.

3. The model may be used to determine the advisability of further ground water development in certain areas.

4. The model may be used for general management of the ground water basin by regulatory agencies.

5. The model should be adaptable to other basins having similar circumstances.

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